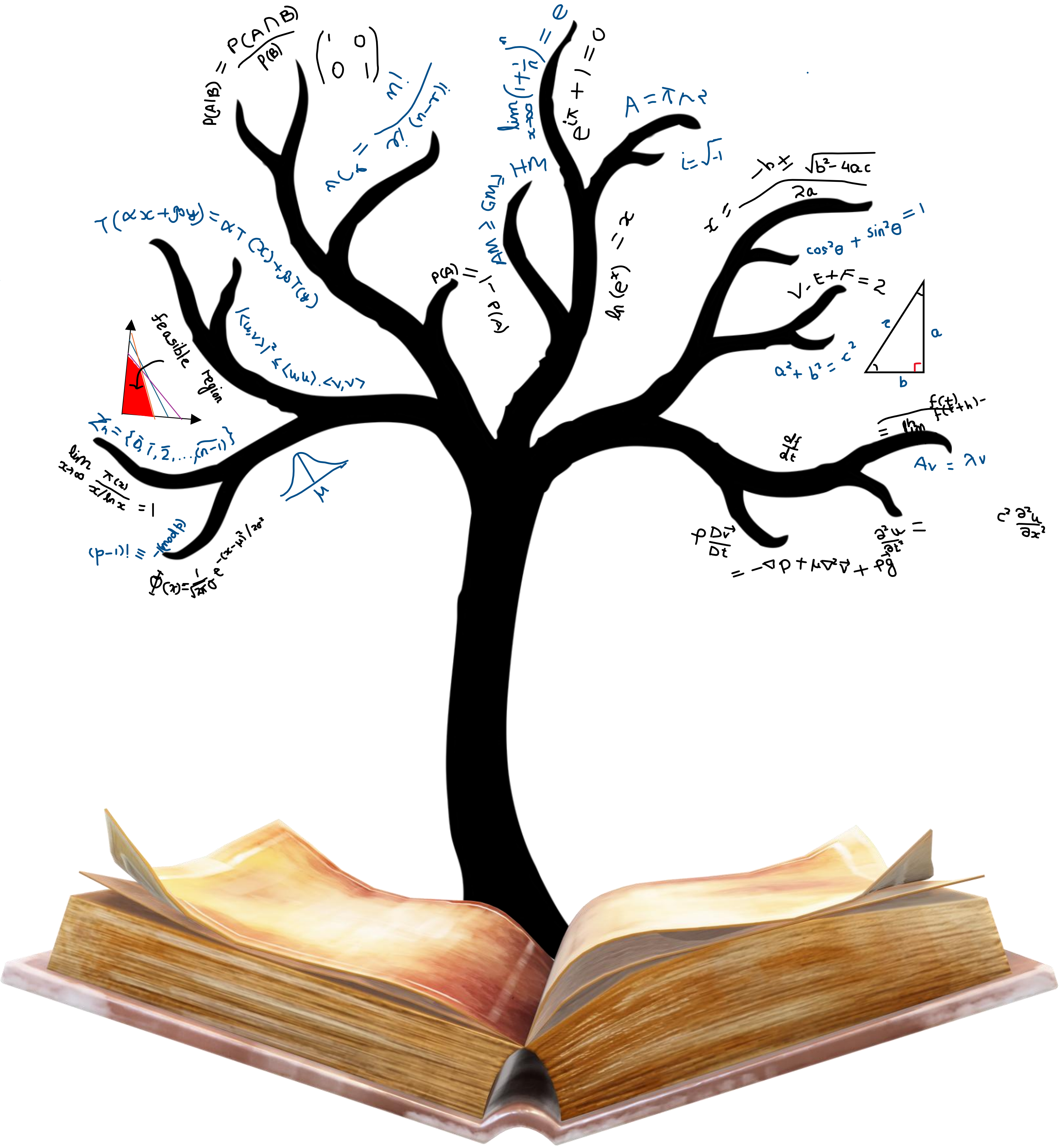
$\int f(x)dx$

NITM Mathematical Bi-monthly

June-July 2025
Issue No. 6

Department of Mathematics,
National Institute of Technology Meghalaya



P(A|B) = P(A∩B)/P(B)
A = πr²
i = √-1
cos²θ + sin²θ = 1
V - E + F = 2
a² + b² = c²
Av = λv
feasible region

“He who hasn't tasted bitter things hasn't earned sweet things.”

— Gottfried Wilhelm Leibniz

(1 July 1646 - 14 November 1716)

DIRECTOR'S MESSAGE

Dear Students, Faculty, and Readers,

I am immensely pleased to introduce the 6th issue of the Department of Mathematics' bimonthly magazine. This magazine represents a significant step forward in creating a platform where the department can showcase our students and faculty members' intellectual curiosity, talent, and dedication.

Mathematics is not just a subject confined to classrooms and textbooks; it is a dynamic and evolving field with the power to shape the world around us. I am proud of the department's commitment to fostering academic excellence and a spirit of innovation.

This magazine is a testament to that pursuit of knowledge. It will serve as a medium for not only disseminating new ideas and research but also for encouraging discussions, collaborations, and creativity within our vibrant mathematical community. I encourage each of you—students and faculty alike—to contribute actively to the growth of this magazine and make it a reflection of our collective brilliance.

As we move forward, let us continue to strive for academic distinction, intellectual curiosity, and a passion for solving the complex problems that mathematics presents. The journey is as important as the destination. I believe that together, we will continue to make strides toward a brighter future for the department and the world of mathematics.

I congratulate the editorial team on their hard work in bringing this publication to life, and I look forward to seeing the magazine evolve in the years to come.

With best wishes,
Prof. Pinakeswar Mahanta
Director, NIT

HoD's Message

Dear esteemed readers,

Greetings from Department of Mathematics, NIT Meghalaya!

It gives me immense pleasure and sense of honor to write in the 6th issue of our departmental magazine "NITM Mathematical". The magazine provides a platform to present various accomplishments and activities of the department on a bimonthly basis and serves as a channel for encouraging creativity, sharing knowledge, ideas, research activities and insights of our departmental family.

The Department of Mathematics started in 2012 with the inception of NIT Meghalaya in Shillong. The Department currently offers two years M.Sc. program and Ph.D. program in addition to catering the mathematical support to other departments of the institute. At present, the department has six faculty members with strong academic and diverse research backgrounds. Ever since its inception, our department, has been striving to maintain excellence in teaching and research providing solid foundation in Mathematics to our students and accomplishing quality research output. Moving ahead, we aim to be a center of excellence for learning Mathematics globally, with special focus on supporting the mathematical requirements in the regional level.

Our department is committed to be vibrant and is dedicated to the holistic development of our students. The creation of this magazine stems from a collective desire to share our thoughts, accomplishments, and aspirations. Working together as a team to ensure its successful publication brings immense delight and it is a privilege to be a part of this process.

I express my sincere gratitude to the editorial board, and everyone who have contributed to this issue of the magazine. I extend my best wishes and sincerely hope that this tradition of the departmental magazine continues for generations to come, fostering happiness, unity, and intellectual growth.

Warm regards,

Dr. Tikaram Subedi

Associate Professor, HoD, MA

Editor's Message

The only way to learn mathematics is to do mathematics. — Paul R. Halmos.

This profound statement not only serves as a guiding principle but also emphasizes the importance of active engagement in mathematics. It brings me great joy to inform you that starting from August 2024, the Department of Mathematics at the National Institute of Technology Meghalaya is introducing its very own publication, the "***NITM Mathematical Bi-monthly.***"

This publication is a collective endeavor by our students and faculty members, designed to ignite a love for mathematics and offer a stage for students to share their insights. Magazines transform the creative potential of our students into tangible contributions, allowing them to identify and showcase their talents through writing. Through this magazine, we aspire to highlight contributions, departmental events, achievements, and the scholarly work of both faculty and students. I encourage all students to participate by submitting interesting mathematical problems, engaging puzzles, stories, and intriguing facts about mathematicians.

I want to express my deepest appreciation to the editorial team—Bankit, Sanchita, Mehjebin, and Dibyasman—for their tremendous dedication and hard work in making this magazine a reality in such a brief period. Our minds are filled with boundless curiosity, and we are continually striving to explore beyond the known. I wish all our students' immense success as they delve into the magazine's contents and set out on fresh intellectual journeys. May this initiative inspire us all to deepen our grasp of mathematics with steadfast determination.

Thank you, and best wishes.

Dr. Timir Karmakar
Assistant Professor
Department of Mathematics

Featured articles

The Detective of Königsberg: A Mathematical Revolution

Himangshu Barman, Research Scholar

Imagine we are living in the 18th century Prussian city of Königsberg. The city is split into four parts by the Pregel River, and these parts are connected by seven picturesque bridges [1].

The challenge? Cross every bridge exactly once in a single walk.

For years, people tried and failed. It seemed impossible, but no one could explain why, until a Swiss mathematician decided to treat the puzzle not as a geography problem, but as a brand-new kind of geometry. That mathematician was Leonhard Euler (pronounced Oiler).

Fig 1: Leonhard Euler (1707–1783), the Swiss mathematician.

To prove the problem unsolvable, he utilized a process of abstraction. He replaced each of the four land areas with a point (vertex) and each of the seven bridges with a line (edge) joining the corresponding points. This transformation created what is formally known in modern terms as a graph or, more specifically, a multigraph, because multiple lines connect the same pair of points [2].

Fig. 2: represents Euler's revolutionary transition from geography to abstract mathematics [2].

Euler focused on the degree of each vertex simply, the number of lines connected to a point. By analyzing the above graph, he established the formal criterion for traversability: a graph is traversable if and only if it is connected and every point is incident with an even number of lines (even degree). Since the graph in Fig. 2 contains points with odd degrees (points A, B, C, and D all have an odd number of incident lines), Euler concluded that the proposed walk was mathematically impossible.

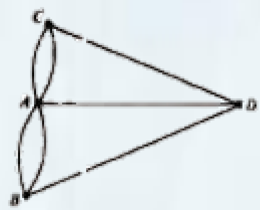


Fig 3: represents the graph of Königsberg bridge problem [2]

We still use Euler's principles today to solve problems vastly more complex than the Königsberg walk:

(1) The World Wide Web: The largest graph in existence. Vertices are websites; edges are hyperlinks [3].
(2) Logistics: Optimization algorithms use graphs to find the most efficient paths between thousands of delivery nodes [3].
(3) The Connectome: Scientists mapping the human brain treat neurons as vertices and synapses as edges to understand how we think.

Reference

(1) Biggs, N. L., Lloyd, E. K., Wilson, R. J., Graph Theory, 1736-1936, Oxford University Press, (1986).
(2) Harary, F., Graph Theory, Addison-Wesley, Reading, MA, (1969).
(3) Newman, M. E. J., Networks: An Introduction, Oxford University Press, (2010).

An Overview of the Theory of Partitions

Dibyasman Sarma, Research Scholar

An equation means nothing to me unless it expresses a thought of God.

– Srinivasa Ramanujan

Introduction

Partition theory is a fascinating and intricate branch of number theory that delves into the ways in which a positive integer can be expressed as a sum of positive integers, known as partitions. The study of partitions is not only a classical subject with deep historical roots but also a vibrant area of contemporary research with wide-ranging applications in mathematics. The origins of partition theory date back to the work of the 18th-century mathematician Leonhard Euler, who first systematically explored partitions and introduced the generating function technique, a powerful analytical tool that remains central to the field. Euler's work laid the foundation for partition theory, which was further developed in the 19th and 20th centuries by mathematicians such as Percy MacMahon, G.H. Hardy, and Srinivasa Ramanujan. Ramanujan, in particular, made groundbreaking contributions with his discovery of the asymptotic formula for the partition function $p(n)$ which approximates the number of partitions of a large integer n, which marks a milestone in analytic number theory.

Mathematical Definition and some Examples

A ***partition*** of a positive integer n is a finite nonincreasing sequence of positive integers $\lambda_1, \lambda_2, \dots, \lambda_r$, such that $\sum_{i=1}^{r} \lambda_i = n$. The λ_i are called the ***parts*** of the partition. The ***partition function*** $p(n)$ is the number of partitions of n. Obviously, $p(n) = 0$ when n is negative. We shall set $p(0) = 1$ with the observation that the empty sequence forms the only partition of zero. The following list presents the next five values of $p(n)$ and tabulates the actual partitions.

Table 1

$p(1) = 1$	1
$p(2) = 2$	$2, 1+1$
$p(3) = 3$	$3, 2+1, 1+1+1$
$p(4) = 5$	$4, 3+1, 2+2, 2+1+1, 1+1+1+1$
$p(5) = 7$	$5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, 1+1+1+1+1$

In the following we can see how the partition function increases quite rapidly with n.

$$p(10) = 42,$$

$$p(20) = 627,$$

$$p(50) = 204226,$$

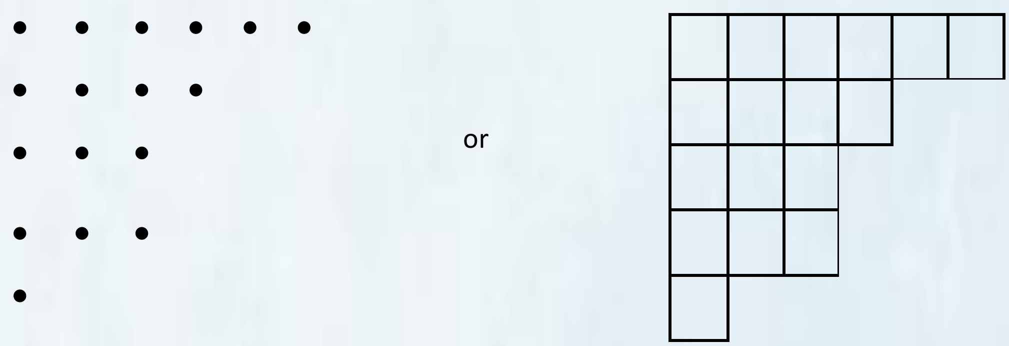
$$p(100) = 190569292,$$

$$p(200) = 3972999029388.$$

Graphical Representation of Partitions

Partitions can be represented visually by Ferrers diagrams (or Young diagrams), consisting of left-justified rows of dots or boxes, with λ_i dots (or boxes) in the i-th row. These diagrams provide powerful combinatorial insight and play a crucial role in representation theory.

The graphical representation of the partition $6 + 4 + 3 + 3 + 1$ is



Euler's Generating Function

In the 18th century, Leonhard Euler transformed the study of partitions from a tedious counting problem into a sophisticated branch of analysis. His primary weapon was the ***generating function***, which is a formal power series used to encode a sequence of numbers so that algebraic manipulations of the series reflect properties of the sequence. Formally, for a sequence $\{a_n\}_{n\geq 0}$, its (ordinary) generating function is

$$f(x) = \sum_{n=0}^{\infty} a_n x^n,$$

where x is an indeterminate. Euler showed that the partition function is the coefficient of x^n in the expansion of:

$$\sum_{n=0}^{\infty} p(n)x^n = \prod_{m=1}^{\infty} \frac{1}{1-x^m}, |x| < 1.$$

Euler’s generating function provides a bridge between combinatorial counting and algebraic manipulation. It allows one to derive identities, recurrence relations, and congruences satisfied by the partition function. For example, the expansion

$$\prod_{m=1}^{\infty} \frac{1}{1-x^m} = 1 + x + 2x^2 + 3x^3 + 5x^4 + \cdots$$

shows that $p(1) = 1, p(2) = 2, p(3) = 3, p(4) = 5$, and so on.

The Hardy-Ramanujan Asymptotic Formula

Godfrey Harold Hardy (left) and Srinivasa Ramanujan (right)

The Hardy–Ramanujan Asymptotic Formula is one of the most stunning achievements in analytic number theory. It provides a way to estimate the value of the partition function $p(n)$ as a sum of positive integers without actually listing them. As n grows, $p(n)$ increases at an incredible rate, making manual counting impossible. In 1918, G.H. Hardy and Srinivasa Ramanujan published a formula that captures this growth with remarkable precision. They discovered that for large n,

$$p(n) \sim \frac{1}{4n\sqrt{3}} exp\left(\pi\sqrt{\frac{2n}{3}}\right).$$

However, the Hardy-Ramanujan formula is an asymptotic (an approximation that gets better as n gets larger), it wasn't an exact formula. In 1937, Hans Rademacher refined their work into a convergent series that gives the exact value of $p(n)$ for any n:

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A_k(n)\sqrt{k} \frac{d}{dn}\left(\frac{sinh\left(\frac{\pi}{k}\sqrt{\frac{2}{3}}\left(n-\frac{1}{24}\right)\right)}{\sqrt{n-\frac{1}{24}}}\right),$$

where $A_k(n)$ is the ***Kloosterman Sum***, which is a complex sum involving roots of unity.

Ramanujan’s Congruences

While the Hardy–Ramanujan and Rademacher formulas focus on the size of the partition function, Ramanujan’s Congruences reveal its hidden rhythmic properties. In 1919, while looking at tables of the first few hundred values of $p(n)$, Srinivasa Ramanujan noticed that the number of ways to partition an integer isn't random, it follows strict rules of divisibility by the primes 5, 7, and 11. He discovered that

1) $p(5n + 4) \equiv 0 \ (mod\ 5)$,
2) $p(7n + 5) \equiv 0 \ (mod\ 7)$ and
3) $p(11n + 6) \equiv 0 \ (mod\ 11)$.

Ramanujan’s congruences are often called the "most beautiful" results in elementary number theory because they connect simple addition to the deep, complex world of modular symmetries.

Conclusion

The theory of partitions is a rich and evolving subject, combining elementary combinatorial ideas with deep analytic and algebraic techniques. It has wide applications in various fields of mathematics, for instance, in representation theory, partitions are far more than just "sums of integers", they serve as the fundamental labeling system for the irreducible representations of some of the most important groups, particularly the Symmetric Group (S_n) and the General Linear Group (GL_n). The partition function and its congruences also has applications beyond mathematics too, specifically, they are essential in Comuper Science, Statistical Mechanics and String Theory. As mathematicians continue to explore and expand the field, partition theory remains a dynamic and fertile area of study, with the potential to unlock new discoveries and applications for years to come.

References

[1] G. E. Andrew, The theory of partitions, Addison-Wesley, 1976.
[2] B. Kumar and R. R. Jha, A study on partition theory, Journal of Mathematical Problems, Equations and Statistics, 2023, 4(1), 156-159.

Research Publications

1. Aritra Narayan Hisabia and Manideepa Saha, Sign patterns of semimonotone matrices, ***Linear Algebra and its Applications***, Vol 724 (2025), 206-217.

Achievement

Sanchita Pramanik was honored with the **Best Presentation Award** for presenting the paper titled *“Couette-Poiseuille flow in a multilayered channel partially filled with a homogenous anisotropic porous layer: Understanding shear stress distribution on glycocalyx”* co-authored with **Timir Karmakar** at the **International Conference on Mathematical Sciences and Computing-Innovations and Applications** organized by the North Eastern Regional Institute of Science and Technology in association with the National Institute of Technology Uttarakhand held from **26th–28th June 2025**.

Solutions to the Problems presented in the 5th issue

Problem 1. For the beta function, prove with the usual meaning that

$$\sum_{n=1}^{\infty}\big(\beta(2,n)-\beta(3,n)\big)=\frac{1}{2}, \quad n\in\mathbb{N}.$$

Solution: (Solved by Subham Saha, Research Scholar)

Recall that for all $m,n\geq 0$, $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ and $\Gamma(n)=(n-1)!$

$$\beta(2,n)-\beta(3,n)=\frac{1!\,(n-1)!}{(n+1)!}-\frac{2!\,(n-1)!}{(n+2)!}$$

$$=\frac{1}{n(n+1)}-\frac{2}{n(n+1)(n+2)}$$

$$=\frac{(n+2)-2}{n(n+1)(n+2)}$$

$$=\frac{1}{(n+1)(n+2)}$$

$$=\frac{1}{n+1}-\frac{1}{n+2}$$

Therefore, $\sum_{n=1}^{\infty}((\beta(2,n)-\beta(3,n))=\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)=\lim_{n\to\infty}\left(\frac{1}{2}-\frac{1}{n+2}\right)=\frac{1}{2}$. ■

Problem 2. For the beta function, prove with usual meaning that

$$\beta(k,n)=\frac{(k-1)\beta(k-1,n)}{(n+k-1)}, \quad k,n\in\mathbb{N}.$$

Solution: (Solved by Subham Saha, Research Scholar)

For all $m,n\in\mathbb{N}$, $\beta(m+1,n)=\int_0^1 x^m(1-x)^{n-1}\,dx$.

Applying integration by parts,

$$\beta(m+1,n) = \left[\frac{x^m(1-x)^n}{-n}\right]_0^1 + \frac{m}{n}\int_0^1 x^{m-1}(1-x)^n\,dx$$

$$= \frac{m}{n}\int_0^1 (1-x).\,x^{m-1}(1-x)^{n-1}\,dx$$

$$= \frac{m}{n}\int_0^1 x^{m-1}(1-x)^{n-1}\,dx - \frac{m}{n}\int_0^1 x^m(1-x)^{n-1}\,dx$$

$$= \frac{m}{n}\,\beta(m,n) - \frac{m}{n}\beta(m+1,n).$$

This gives,

$$\frac{m+n}{n}\beta(m+1,n) = \frac{m}{n}\beta(m,n).$$

Hence,

$$\beta(m+1,n) = \frac{m}{m+n}\beta(m,n).$$

Replacing m by $k-1$ in the above relation we get the desired result. ■

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