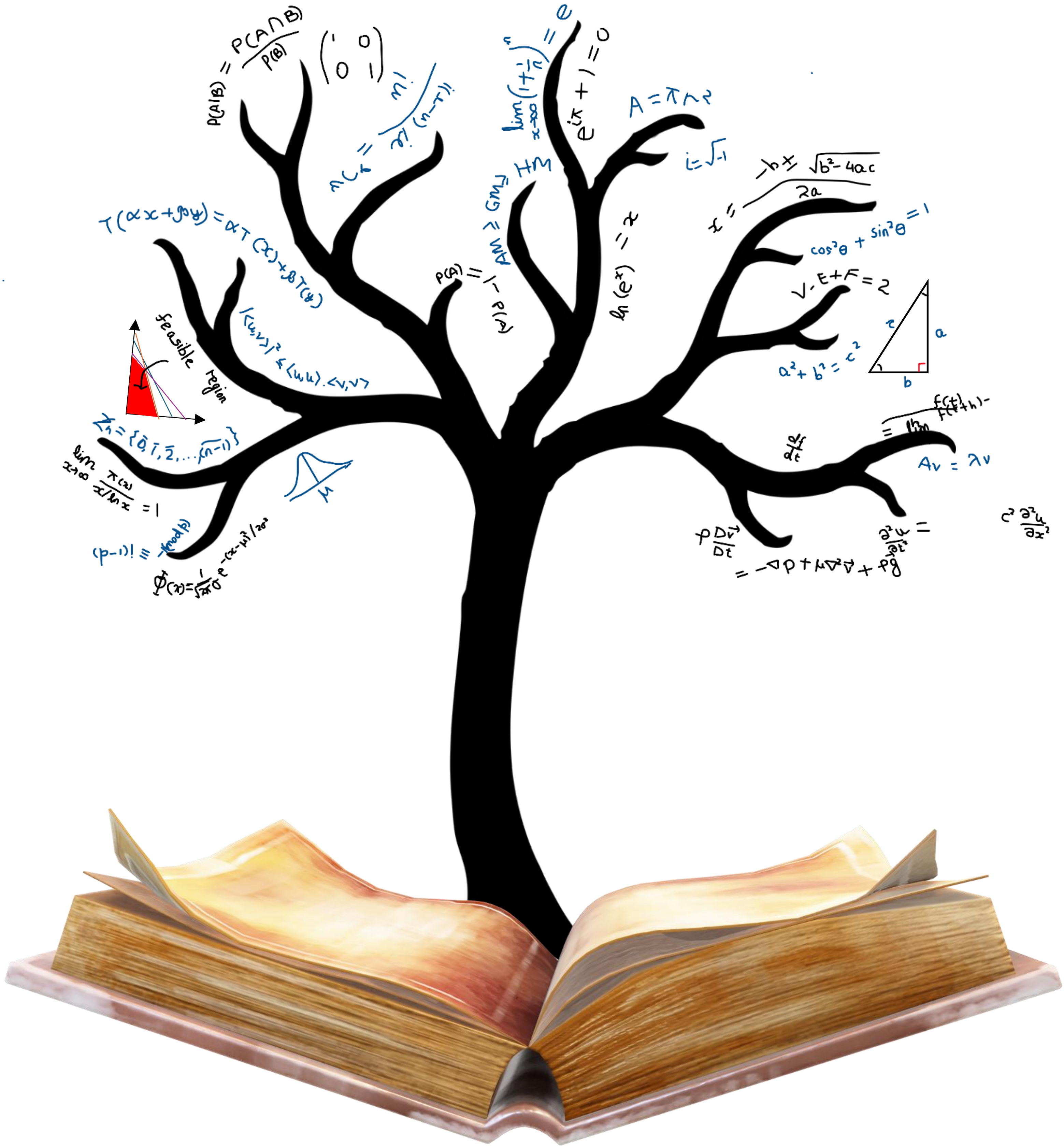
NITM Mathematical Bi-monthly

February-March 2025

Issue No. 4

Department of Mathematics,
National Institute of Technology Meghalaya


P(A|B) = P(A∩B)/P(B)
feasible region
V - E + F = 2
a² + b² = c²
A = πr²
i = √-1
e^{iπ} + 1 = 0
cos²θ + sin²θ = 1
Av = λv
AM ≥ GM ≥ HM
ln(e^x) = x
P(A') = 1 - P(A)

“In mathematics as in other fields, to find one self lost in wonder at some manifestation is frequently the half of a new discovery.”

— Peter Gustav Lejeune Dirichlet

(13 February 1805 - 5 May 1859)

DIRECTOR'S MESSAGE

Dear Students, Faculty, and Readers,

I am immensely pleased to introduce the 4th edition of the Department of Mathematics' bimonthly magazine. This magazine represents a significant step forward in creating a platform where the department can showcase our students and faculty members' intellectual curiosity, talent, and dedication.

Mathematics is not just a subject confined to classrooms and textbooks; it is a dynamic and evolving field with the power to shape the world around us. I am proud of the department's commitment to fostering academic excellence and a spirit of innovation.

This magazine is a testament to that pursuit of knowledge. It will serve as a medium for not only disseminating new ideas and research but also for encouraging discussions, collaborations, and creativity within our vibrant mathematical community. I encourage each of you—students and faculty alike—to contribute actively to the growth of this magazine and make it a reflection of our collective brilliance.

As we move forward, let us continue to strive for academic distinction, intellectual curiosity, and a passion for solving the complex problems that mathematics presents. The journey is as important as the destination. I believe that together, we will continue to make strides toward a brighter future for the department and the world of mathematics.

I congratulate the editorial team on their hard work in bringing this publication to life, and I look forward to seeing the magazine evolve in the years to come.

With best wishes,
Prof. Pinakeswar Mahanta
Director, NIT

HoD's Message

As the Head of the Department of Mathematics (MA), it is truly an honor for me to write for the 4th issue of our departmental magazine. This platform allows our faculty, staff, and students to showcase their achievements, express their opinions, and explore new mathematical concepts. I believe that this magazine will become an essential channel for sharing knowledge, ideas, and research insights that will inspire us all.

The Department of Mathematics, which started functioning in June 2012, currently offers 2-year M.Sc. and PhD programs. M.Sc. students are selected based on their ranking through CCMN and the institution mode, while PhD students are selected through interviews based on their GATE/NET scores. In addition to our core programs, the department also plays a vital role as a supporting pillar for various B. Tech and M. Tech programs within the institute.

The creation of this magazine stems from a collective desire to share our thoughts, accomplishments, and aspirations. Working together as a team to ensure its successful publication brings immense delight. I feel privileged to be part of this process and am filled with joy in nurturing our students, contributing to society, and fostering academic excellence.

My team and I remain dedicated to the holistic development of our students within the institute. I extend my best wishes to all MA family members and sincerely hope that this tradition of the departmental magazine continues for generations to come, fostering happiness, unity, and intellectual growth.

Warm regards,
Dr. Adarsha Kumar Jena
Assistant Professor, HoD, MA

Editor’s Message

The only way to learn mathematics is to do mathematics. — Paul R. Halmos.

This profound statement not only serves as a guiding principle but also emphasizes the importance of active engagement in mathematics. It brings me great joy to inform you that starting from August 2024, the Department of Mathematics at the National Institute of Technology Meghalaya is introducing its very own publication, the “***NITM Mathematical Bi-monthly.***”

This publication is a collective endeavor by our students and faculty members, designed to ignite a love for mathematics and offer a stage for students to share their insights. Magazines transform the creative potential of our students into tangible contributions, allowing them to identify and showcase their talents through writing. Through this magazine, we aspire to highlight contributions, departmental events, achievements, and the scholarly work of both faculty and students. I encourage all students to participate by submitting interesting mathematical problems, engaging puzzles, stories, and intriguing facts about mathematicians.

I want to express my deepest appreciation to the editorial team—Bankit, Sanchita, Dixita, and Dibyasman—for their tremendous dedication and hard work in making this magazine a reality in such a brief period. Our minds are filled with boundless curiosity, and we are continually striving to explore beyond the known. I wish all our students’ immense success as they delve into the magazine's contents and set out on fresh intellectual journeys. May this initiative inspire us all to deepen our grasp of mathematics with steadfast determination.

Thank you, and best wishes.

Dr. Timir Karmakar
Assistant Professor
Department of Mathematics

Featured articles

Newton's Approximation of π

Souvik Bhattacharya; M. Sc (Batch 2023-2025)

Long before calculators—and even before calculus had a name—Sir Isaac Newton was using infinite series with astonishing skill. In his *Methodus Fluxionum et Serierum Infinitarum* (written in 1671 but published decades later), Newton crafted an ingenious method to approximate π. What makes his approach remarkable is the way he unites geometry, series, and integration in a single elegant argument—anticipating techniques that modern calculus would formalize much later.

Newton has certainly mastered the concepts of analytic geometry, and cast his work in this format. He began with a semicircle having its center C at (1/2, 0) and radius r = ½ as shown in the figure. He knew that the circle's equation was

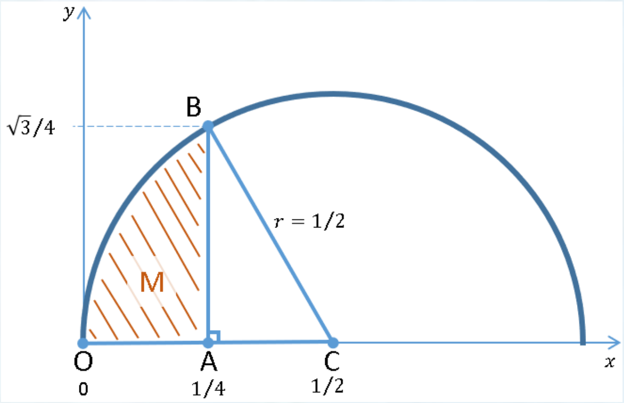
$$\left(x-\frac{1}{2}\right)^2+y^2=\left(\frac{1}{2}\right)^2$$

Simplifying and solving for y gives the equation of the upper semicircle as

$$y=\sqrt{x-x^2}=x^{\frac{1}{2}}(1-x)^{\frac{1}{2}}$$

exactly why he chose this particular semicircle may seem a complete mystery, but its special utility will become clear in the end. Replacing $(1-x)^{\frac{1}{2}}$ by its binomial expansion, we have

$$y\ =x^{\frac{1}{2}}-\frac{1}{2}x^{\frac{3}{2}}-\frac{1}{8}x^{\frac{5}{2}}-\frac{1}{16}x^{\frac{7}{2}}-\frac{5}{128}x^{\frac{9}{2}}-\frac{7}{256}x^{\frac{11}{2}}-\ \dots\dots\dots\dots$$



And now, the genius of Isaac Newton becomes apparent. He let A be the point (1/4, 0) as indicated in the figure, and he drew AB perpendicular to the semicircle's diameter. He then attacked the shaded area OBA in two different ways:

1. **Area by fluxions**: As we have seen, Newton knew how to find the area under such a curve from its starting point at 0 rightward to the point x=1/4. That is, by Rules 1 and 2 of De Analysi, the shaded area was just

$$\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}\left(\frac{2}{5}x^{\frac{5}{2}}\right) - \frac{1}{8}\left(\frac{2}{7}x^{\frac{7}{2}}\right) - \frac{1}{16}\left(\frac{2}{9}x^{\frac{9}{2}}\right) - \ \dots = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{5}x^{\frac{5}{2}} - \frac{1}{28}x^{\frac{7}{2}} - \frac{1}{72}x^{\frac{9}{2}} - \frac{5}{704}x^{\frac{11}{2}} - \ \dots \qquad (**)$$

   Evaluated for the value x = ¼. the genius of his approach is that the resulting expression simplifies beautifully when we evaluate it, since

$$\left(\frac{1}{4}\right)^{3/2} = \left(\sqrt{\frac{1}{4}}\right)^3 = \frac{1}{8}, \left(\frac{1}{4}\right)^{5/2} = \left(\sqrt{\frac{1}{4}}\right)^5 = \frac{1}{32}$$

   and so on.

   Thus, we approximate the shaded area (OBA), using the first nine terms of series (**), by

$$\frac{1}{12} - \frac{1}{160} - \frac{1}{3584} - \frac{1}{36864} - \frac{5}{1441792} - \ \dots - \frac{429}{163208757248} = 0.07677310678$$

2. **Area by geometry:** Newton next reexamined the problem of the shaded area from a purely geometric perspective. He first determined the area of the right triangle △BAC. Notice that the length of AC is ¼, while CB, being a radius, has length r = ½. A direct application of the Pythagorean theorem yielded

$$\text{AB} = \sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2} = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4}$$

   Hence,

$$\text{Area}(\triangle \text{BAC}) = \tfrac{1}{2}\,(\text{AC})(\text{AB}) = \frac{\sqrt{3}}{32}$$

So far, so good. Next Newton wanted the area of the wedge or ple- shaped sector OCB. To determine this area, he again referred to △BAC. With the length of AC being exactly half that of the hypotenuse CB, he recognized this as a familiar 30°, 60°, 90° right angle; in particular angle ACB was a 60° angle.

Again, one is struck by his penetrating insight, for by placing his perpendicular at a point other than A, he would have emerged with a simple 60° angle when he needed it most. But, knowing that the angle of the sector was 60° - that is, one-third of the 180° angle forming the semicircle Newton could see that the area of the sector was likewise a third of the area of the semicircle. In short,

$$\text{Area(sector)} = 1/3\ \text{Area(semicircle)} = \frac{1}{3}\left(\frac{1}{2}\pi r^2\right) = \frac{1}{3}\left[\frac{1}{2}\pi\left(\frac{1}{2}\right)^2\right] = \frac{\pi}{24}$$

The perceptive reader, recalling that this great theorem was to have been Newton's approximation of π , may have been worrying about how and when this constant was ever going to enter the argument. At last, π has appeared in Newton's chain of reasoning, and there now remains just a final step or two to get a wonderfully efficient approximation of it. Thus, the geometric approach to the shaded area yields

$$\text{Area (OBA)} = \text{Area(sector)} - \text{Area}(\triangle \text{BAC}) = \frac{\pi}{24} - \frac{\sqrt{3}}{32}$$

Equating this result with the fluxion/binomial theorem approximation for the same shaded area above, we have $0.07677310678 \approx \frac{\pi}{24} - \frac{\sqrt{3}}{32}$ and solving this for π gives us the estimate

$$\pi \approx 24\left(0.07677310678 + \frac{\sqrt{3}}{32}\right) = 3.141592668 \ldots$$

The amazing thing about this estimate is that, with just nine terms of the binomial expansion, we have found π correct to seven decimal places, and our estimate above differs from the true value of π by less than 0.000000014.

Newton's approach to approximating π is a brilliant example of mathematical creativity—merging algebra, infinite series, and geometry in a way centuries ahead of its time. With just a semicircle, a well-chosen point, and a series expansion, he reached a result that still stuns today. It's a quiet testament to Newton's genius—and a reminder that deep insight often lies in simple shapes.

Playing with Numbers: From Chaos to 6174

Subham Saha, Research Scholar

"Mathematics, rightly viewed, possesses not only truth but supreme beauty."
— Bertrand Russell

Let's play a quick number game before we get started. Let's start with the present year, 2025, which is a number we are rather familiar with.

We'll follow a process. Rearrange the digits in **descending order**, and then in **ascending order**, subtract the smaller from the larger, and repeat.

1. 2025 → Descending: 5220 | Ascending: 0225 → 5220 − 0225 = **4995**
2. 4995 → Descending: 9954 | Ascending: 4599 → 9954 − 4599 = **5355**
3. 5355 → Descending: 5553 | Ascending: 3555 → 5553 − 3555 = **1998**
4. 1998 → Descending: 9981 | Ascending: 1899 → 9981 − 1899 = **8082**
5. 8082 → Descending: 8820 | Ascending: 0288 → 8820 − 0288 = **8532**
6. 8532 → Descending: 8532 | Ascending: 2358 → 8532 − 2358 = **6174**

We've landed on the number **6174**.
We will obtain the same result if we repeat the same step:7641−1467 = **6174**.

Now that the process has stabilized—it won't alter!

Now let's try another number: **1947**, the year India became independent.

1. 1947 → Descending: 9741 | Ascending: 1479 → 9741 − 1479 = **8262**
2. 8262 → Descending: 8622 | Ascending: 2268 → 8622 − 2268 = **6354**
3. 6354 → Descending: 6543 | Ascending: 3456 → 6543 − 3456 = **3087**
4. 3087 → Descending: 8730 | Ascending: 0378 → 8730 − 0378 = **8352**
5. 8352 → Descending: 8532 | Ascending: 2358 → 8532 − 2358 = **6174**

Again, we reach **6174**! Isn't it fascinating?

What we just observed is not a trick or a lucky coincidence. It's a reliable, predictable pattern that works across a vast set of four-digit numbers. The concept was introduced by Indian mathematician **D.R. Kaprekar** in **1949**, whose work in number theory continues to be of interest. And the number **6174** is called **Kaprekar's constant.**

Who Was D. R. Kaprekar?

Dattatreya Ramchandra Kaprekar (1905–1986) was an Indian mathematician who made significant contributions to number theory, notably in the field of recreational mathematics, despite never holding a formal research position in a university. Born in **Dahanu**, a town in **Maharashtra**, Kaprekar showed a fascination with numbers from an early age. He graduated from Fergusson College in Pune in **1929** and later became a school teacher. While he didn't work in an academic research institute, he pursued mathematics out of sheer passion and curiosity.

Kaprekar is sometimes called the **"man who loved numbers"**, and rightly so. His approach to mathematics was unorthodox but playful and deeply insightful. Instead of abstract theory, he was driven by patterns and numerical oddities—things most traditional mathematicians often overlooked.

Initially, his work was dismissed by mainstream mathematicians. But over the time, his work got recognized. In fact, British mathematician **Martin Gardner**, famous for popularizing mathematical puzzles through ***Scientific American***, praised Kaprekar's discoveries and helped introduce his work to a global audience. Some of his key contributions are **Kaprekar's Constant**, **Kaprekar Numbers**, **Self-numbers or Colombian numbers**, **Harshad Numbers.**

The Kaprekar Routine:

Let's now return to the process, which we call the Kaprekar Routine.

Take any four-digit number (**not a multiple of 1111**), and repeatedly rearrange its digits in descending and ascending order, subtract the smaller from the larger, and repeat the process—you will always end up at **6174**. And once you reach it, you stay there forever. No matter where you start, you will reach **6174** in **at most seven steps**.

Interestingly, a similar process also works for 3-digit numbers. In this case, the process leads to a different constant **495**.

Though it may seem like a number puzzle, these constants touch on deeper mathematical ideas, such as iteration, digit manipulation and fixed points—concepts relevant even in areas like computer algorithms and theoretical mathematics. Kaprekar’s constant reminds us that even the most ordinary-looking numbers can hold extraordinary secrets.

Kaprekar’s constant is more than a curious trick—it’s a reminder that mathematics is full of surprises hidden in simple places. Behind an ordinary subtraction process lies a profound idea of convergence and symmetry. Kaprekar’s discoveries may not have direct applications in engineering or industry. But they spark ideas in surprising places—like cryptography, digit-based algorithms, pattern recognition, and convergence analysis. They show how even simple number games can lead to deep mathematical thinking. His work reminds us that mathematics isn't always about utility—sometimes, it's about the joy of discovery.

So, next time you're bored, take a number—and let it led you to 6174, the magical constant that always calls you home.

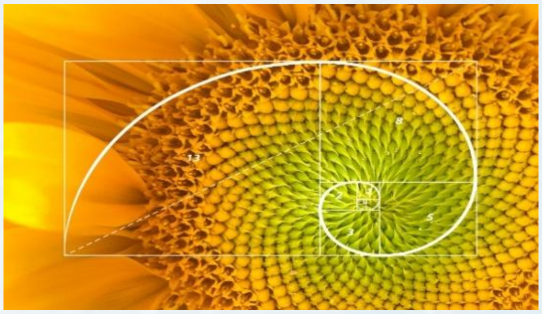
Beauty of Fibonacci Sequence

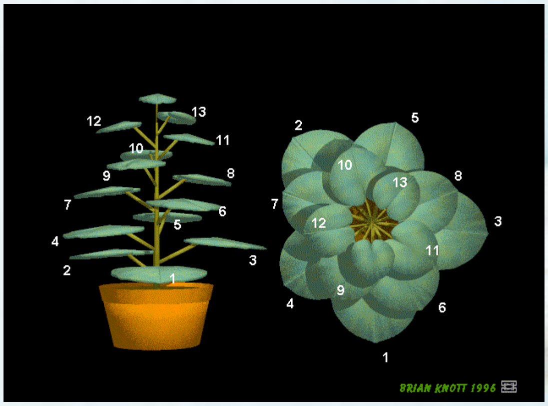
Sanchita Pramanik, Research Scholar

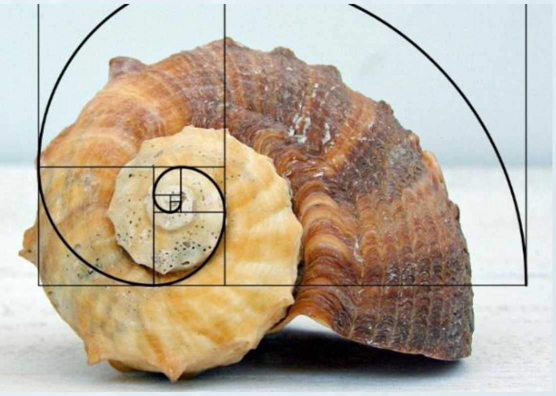
People frequently think of math as a field of logic, accuracy, and computation. But there are patterns in its numbers that are so beautiful they appear to touch the soul. The Fibonacci sequence is one of the most beautiful and logical examples of how beauty and logic can work together. It starts off simple: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34....The total of the two numbers preceding it is the number itself. But underneath this simple surface lies a world of wonder. The Fibonacci sequence seems like a hidden trademark of nature, gently directing development and design. It may be seen in the spirals of galaxies and the curves of seashells.

Fibonacci in Nature: The Fibonacci sequence is amazing because it shows up so often in nature. It looks like nature knows how to flourish and be in harmony with math.

1. The seeds of sunflowers are arranged in spirals, with 34 spirals going one way and 55 going the other.
2. Shells, like the nautilus, frequently follows a logarithmic spiral that is quite similar to the series.
3. Fibonacci numbers show themselves in the spirals of pinecones and pineapples.
4. Fibonacci-related angles typically determine how branches and leaves grow in ways that get the most sunlight.

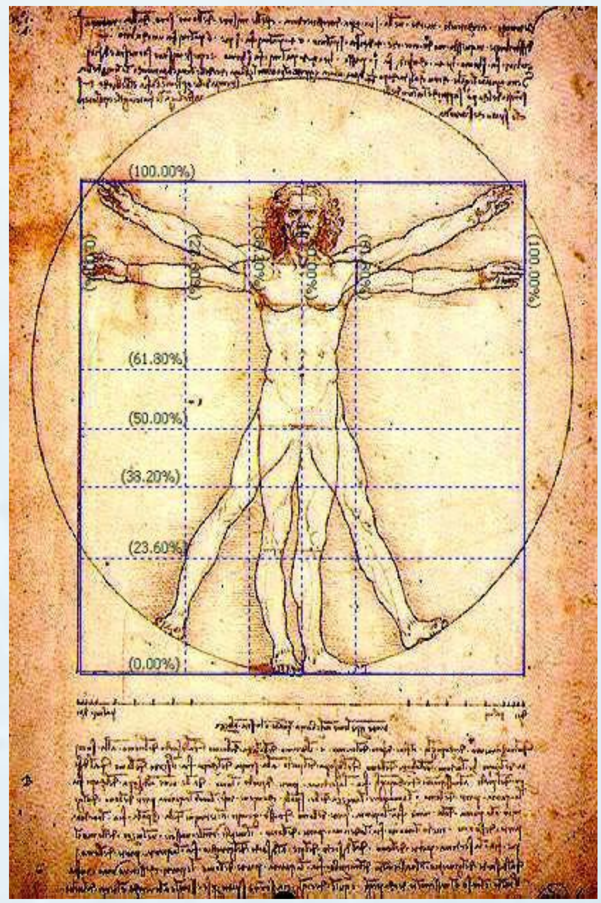






Fibonacci in Art and Architecture: Fibonacci's beauty has always fascinated artists and architects. The ratio of two Fibonacci numbers that come one after the other becomes closer to the Golden Ratio (around 1.618), which has been thought of as the most beautiful number since the time of the Greeks. The Parthenon, Leonardo da Vinci's "Vitruvian Man," and the Great Pyramid of Giza are said to have golden proportions. The Golden Rectangle, which relies on Fibonacci numbers, is still used to direct aesthetics in contemporary logos, paintings, and picture compositions.

Fibonacci in painting



Vitruvian Man

Fibonacci in Music and Poetry: The sequence is not just pretty to look at, it also makes sound move in a certain way. Fibonacci numbers can change rhythms, measures, or scales in music. People think that composers like Bartók and Debussy used these numbers to make their music. Some poets have tried writing Fibonacci poems, where the number of syllables in each line goes like this: 1, 1, 2, 3, 5, 8...

LIFE
is
more
than a
search for X,
some algebraic
equation that can be worked out
with pencil and paper and trusty calculator
Figures cannot account for serendipity, miracles, random acts of kindness
Worthwhile journeys are neither linear nor balanced
They are not problems to be solved,
but roundabout paths
to the truth
within
one's
self

FIB poetry on Fibonacci sequence

The Fibonacci sequence is not only a list of numbers, but also a reminder that beauty, symmetry, and logic may all be true at the same time. The Fibonacci sequence inspires you to look more closely at the universe, whether you are a mathematician, an artist, a scientist, or just someone who is interested. It shows you that even in its most complicated forms, nature dances to the beat of numbers.

Source: All images are taken from google.

Towards a Definition of Integration

Timir Karmakar, Assistant Professor

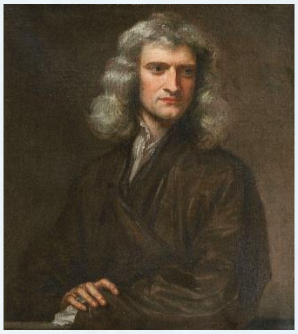
The Fundamental Theorem of Calculus establishes the deep connection between differentiation and integration, showing that they are essentially inverse processes. It is expressed in two complementary parts: one dealing with the differentiation of an integral, and the other with the integration of a derivative. Under suitable hypothesis on the functions f and F, the Fundamental Theorem of Calculus states that

$(i)\ \int_a^b F'(x) = F(b) - F(a)$ and

$(ii)\ if\ G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

Before we begin a rigorous examination of these statements, it is essential to first establish a clear definition of $\int_a^b f$.

Historically, integration was introduced as the inverse process of differentiation. In other words, the integral of a function f was understood to be a function F that satisfies $F' = f$. Newton, Leibniz, Fermat, and other founders of calculus explored this perspective, linking antiderivatives to the problem of computing areas. However, from the standpoint of rigorous analysis, this approach is unsatisfactory because it restricts integration to a relatively small class of functions. Recall that every derivative satisfies the intermediate value property (Darboux's Theorem). Consequently, any function with a jump discontinuity cannot arise as a derivative.



Sir Isaac Newton (1643-1727)

Jean-Gaston Darboux (1842-1917)

Bernhard Riemann (1826-1866)

If we want to define integration via antidifferentiation, then we must accept the consequence that the function is as simple as

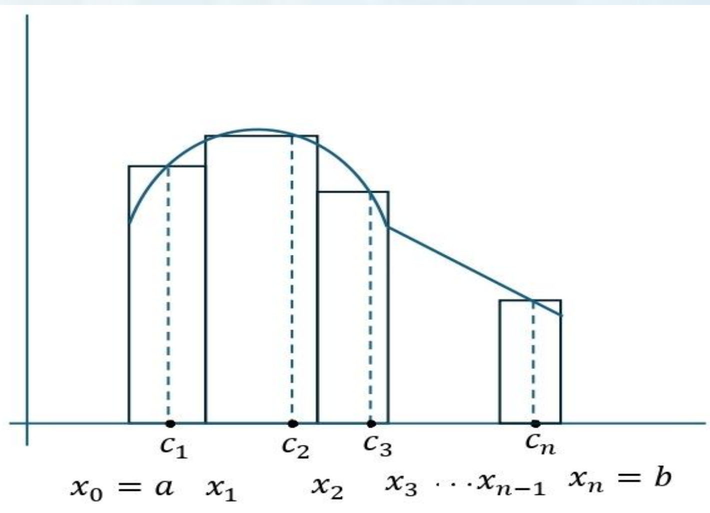
$$h(x) = \begin{cases} 0, & x < 0.5 \\ 1, & x \geq 0.5 \end{cases}$$

is not integrable over the interval $[0,1]$.

Around 1850, a significant shift in perspective emerged in the work of Cauchy and, shortly thereafter, Bernhard Riemann. The central idea was to separate the concept of integration entirely from differentiation and instead develop a rigorous definition of the integral based on the notion of "area under the curve." The reasons for this shift were multifaceted. As noted earlier, the very notion of a *function* undergoing a profound transformation. The traditional understanding of a function as a holistic formula such as $f(x) = x^2$ was being replaced with a more liberal interpretation, which included such bizarre constructions as Dirichlet's function. A major catalyst in this evolution was the emerging theory of Fourier series, which demanded, among other things, a method for integrating these increasingly irregular functions.

The Riemann integral, still the standard approach introduced in elementary calculus courses is the form of integration most studied at the introductory level. Starting with a function f on $[a, b]$, we partition the domain into small subintervals. On each subinterval $[x_{k-1}, x_k]$, we pick some point $c_k \in [x_{k-1}, x_k]$ and use the y-value $f(c_k)$ as an approximation for f on $[x_{k-1}, x_k]$.

Graphically, the idea is to approximate the area between the f and the x-axis using a collection of thin rectangles. The area of each rectangle is given by the Riemann sum $\sum_{k=1}^{n} f(c_k)(x_k - x_{k-1})$. Note that the "area" here comes with the understanding that areas below the x-axis are assigned a negative value.



Riemann Sum

What becomes clear from the graph is that the accuracy of a Riemann sum improves as the rectangles become thinner. In essence, the integral is defined as the limit of these approximating sums as the widths of the subintervals in the partition approach zero. This limit, if it exists, is the Riemann's definition of $\int_a^b f$.

This leads us to several important questions. While giving a precise meaning to the limit described above is not especially difficult, the deeper issue one that also concerned Riemann is determining which functions can actually be integrated by this method. In other words, under what conditions on f does this limit exist?

The theory of the Riemann integral rests on the idea that smaller subintervals yield better approximations to the function f. On each subinterval $[x_{k-1}, x_k]$, the function f is approximated by its value at some point $c_k \in [x_{k-1}, x_k]$. The quality of the approximation is directly related to the difference $|f(x) - f(x_k)|$ as x ranges over the subinterval. Because the subintervals can be chosen to have arbitrarily small width, this means that we want $f(x)$ to be close to $f(c_k)$ whenever x is close to c_k. But this sounds like a discussion of continuity! We will soon see that the continuity of f is intimately related to the existence of the Riemann integral $\int_a^b f$.

Is continuity sufficient to guarantee that Riemann sums converge to a well-defined limit? Is it truly necessary, or can the Riemann integral also accommodate certain discontinuous functions such as $h(x)$ mentioned earlier? Relying on the intuitive notion of area, I would seem that $\int_0^1 h = \frac{1}{2}$. But does the Riemann integral yield this result? More generally, how discontinuous can a function be and remain integrable? For instance, can the Riemann integral make sense of something as extreme as Dirichlet's function on the interval $[0,1]$?

Consider the function

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

This raises another interesting question. Here we have an example of a differentiable function where the derivative $g'(x)$ is not continuous. As we broaden our understanding of integrable functions, it becomes important to reconnect integration with differentiation. Having defined integration independently of differentiation, we would like to come back and investigate the conditions under which the equations (i) and (ii) from the Fundamental Theorem od Calculus stated earlier hold. If we were to make a "wish list" of functions we would like to be integrable, then, in light of equation (i), it is natural to expect that the set of integrable functions should at least include all derivatives. Since derivatives need not be continuous, this motivates the need for an integral that can still accommodate certain discontinuities. To achieve this, instead of relying solely on Riemann sums, we introduce upper sums and lower sums. Instead of taking an ordinary limit, we make use of the notions of *supremum* and *infimum.* Intuitively, a lower sum provides an underestimate of the integral, while an upper sum provides an overestimate. As the partition of the interval is refined, the lower sums increase, and the upper sums decrease. A function is *'integrable'* if the upper and lower sums 'meet' at some common value in the middle.

References

1. Stephen Abbott, "Understanding Analysis", Springer, New York, 2001.
2. Robert G. Bartle, "The Elements of Real Analysis", Second Edition. John Wiley and Sons, New York, 1964.
3. Robert G. Bartle, "Return to the Riemann Integral", American Mathematical Monthly, October 1996.

Editorial Team

	Dr. Timir Karmakar (Assistant Professor)
	Bankitdor M. Nongrum (Research Scholar, P22MA008)
	Dixita Sonowal (Research Scholar, P23MA001)
	Sanchita Pramanik (Research Scholar, P23MA002)
	Dibyasman Sarma (Research Scholar, P22MA007)

CONTACT US

Department of Mathematics

National Institute of Technology Meghalaya

Saitsohpen, Sohra (Cherrapunji)-793108

Meghalaya, India

Phone No: 0364-2501294, FAX: 0364-2501113

Editor's Contact Number: +91 8101058282